THE DETERMINATION OF FIBRE DIAMETER DISTRIBUTION

O. Mayo, B. Crook, J. Lax, A. Swan and T. W. Hancock*
CSIRO Division of Animal Production, Clunies Ross Street, Prospect, NSW 2149
*Department of Plant Science, Waite Agricultural Research Institute,
University of Adelaide

SUMMARY

This paper presents evidence that fleece quality might be improved by selection on the first four moments of the fibre diameter distribution.

INTRODUCTION

The distribution of fibre diameter (FDD) is the complex outcome of developmental processes that are only now beginning to be unravelled (Nagorcka and Adelson, 1992). It is also of importance in determining wool quality, both for the appearance and comfort of the finished garment and for its performance during processing.

The finer the wool, other things being equal, the higher the quality, but if the distribution is too long-tailed to the right, its impact and handle may be impaired. Over many years, its has been shown that clean fleece weight may be increased without increasing fibre diameter by use of an appropriate index, but concerns have occasionally been raised that this may eventually produce a skewed distribution that has an "excessive coarse edge".

It is therefore of importance to understand FDD better.

Gilmour and Atkins (1992) have modelled FDD as a mixture of normal distributions (Pearson, 1894), and concluded that this provided a better description of the distribution yielded by the FFDA machine. They raised three questions:

1. Can the fine and coarse components of the mixture be explained biologically or histologically?
2. Are the two components determined by the same set of genes?
3. How do the components relate to important production traits?

This paper addresses question 3.

DETERMINATION OF THE DISTRIBUTION

Table 1 shows statistics of the FDD for 13 Merino studs sampled in NSW It can be seen that all distributions are positively skewed and strongly leptokurtic.
Across flocks, skewness (Fisher's g_1) depends linearly on mean and variance:

$$\text{skewness} = 1.30\ [\pm\ 0.43] + 0.098\ [\pm\ 0.033]\ \text{mean} - 0.455\ [\pm\ 0.096]\ \text{standard deviation}$$

as also is kurtosis (Fisher's g_2):

$$\text{kurtosis} = 14.42\ [\pm\ 2.21] + 0.299\ [\pm\ 0.177]\ \text{mean} - 3.47\ [\pm\ 0.50]\ \text{standard deviation}$$

(In these regression equations, standard errors of parameter estimates are shown in square brackets).

Skewness and kurtosis are highly correlated ($r = 0.783$, $P < 0.001$). These results are phenotypic, but if it is found that differences in skewness and kurtosis have a genetical basis, there is evidently substantial variability available for selection.

Table 1. Statistics of fibre diameter distribution for 15 NSW Merino studs (Crook unpublished) (FD in micron, SD = standard deviation, 5000 fibres measured on each of 100 animals from each stud)

Stud	Mean	SD of Mean	SD	SD of SD	Skew -ness	SD of Skew -ness	Kur-tosis	SD of Kur-tosis
1	22.07	1.50	4.92	0.79	1.31	0.36	3.91	1.95
2	21.29	1.56	4.55	0.64	1.37	0.34	5.31	3.55
3	19.33	1.27	3.72	0.44	1.55	0.49	7.84	6.46
4	18.03	1.19	3.76	0.69	1.55	0.46	7.43	6.69
5	19.35	1.35	4.01	0.64	1.44	0.41	5.73	4.06
6	20.71	1.42	4.33	0.63	1.33	0.38	5.26	3.34
7	21.31	1.48	4.43	0.67	1.39	0.37	5.30	3.25
8	22.01	1.73	5.33	0.81	0.96	0.42	2.77	1.96
10	18.42	1.09	4.28	0.49	1.09	0.35	4.70	4.14
11	19.63	1.62	4.54	0.67	1.28	0.41	4.99	4.32
15	19.17	1.37	4.22	0.52	1.17	0.32	4.64	3.15
16	20.21	1.34	4.00	0.47	1.40	0.34	5.92	3.68
17	19.83	1.36	4.16	0.58	1.37	0.40	6.56	4.73
19	20.03	1.44	3.87	0.54	1.40	0.50	7.14	6.44
20	18.34	1.25	3.88	0.52	1.19	0.47	6.47	6.67
Mean	19.98		4.267		1.320		5.598	
Sd	1.28		0.444		0.162		1.357	

Before fitting mixture distributions, it is advisable to test for significant departure from normality on an appropriate scale. Mendell, Finch and Thode (1993) have recently shown which tests are appropriate for which mixtures, and should be consulted for this purpose.

Hancock (1978) fitted normal mixtures to a number of traits in two flocks of South Australian Merinos selected for increased clean fleece weight. He did this after testing by the method of Shapiro and Francia (1972). Table 2 shows the results, in terms of numbers of significant tests (at the 5% level) and numbers of satisfactory partitions of the distribution into two normal components. Thirty nine of 192 independent tests were significant, and 22 of the 39 mixtures could be resolved. It is of interest that only 8 of 48 fibre diameter distributions departed significantly from the normal.

Table 2. Numbers of distributions significantly non-normal (bimodal) by the Shapiro-Francia method, and numbers of cases where the method of movements produced a satisfactory separation of the components of the mixture (from Hancock, 1978).

Flock	Index				Visual			
Sex	Female		Male		Female		Male	
Trait	Non-normal	Mixture	Non-normal	Mixture	Non-normal	Mixture	Non-normal	Mixture
Bodyweight	4	2	1	0	3	3	4	3
Clean fleece weight	0	0	1	1	3	2	1	1
Fibre diameter	1	0	3	1	1	1	3	2
Secondary fibre number (SFN)	4	2	4	0	3	2	3	2

It is reasonable to conclude that a mixture frequently provides a reasonable description of the empirical FDD among animals, just as Gilmour and Atkins showed within animals. This does not mean that it is biologically meaningful, especially if Gilmour and Atkins's failure to relate P and S follicles to the coarse and fine mixture components is borne out in practice. In this context, it should be noted that SFN in Table 2 is as frequently non-Gaussian as any other trait, that these results were obtained by R E Brady using stained sections and that the genetic correlation between primary follicle number (PFN) and SFN in these flocks was significantly greater than zero (Hancock, Mayo, Brady and Hooper, 1979).

What process could mimic a mixture, or generate an apparent mixture? One model for fibre formation supposes that a given area of skin can only develop a given area of follicles. On such a model, the frequency of diameter of successive follicles, X, might decline exponentially, above some minimum diameter M, ie, if Z = X-M,

$$f_Z(z) = \theta e^{-\theta z},\ z>0$$

In addition, there could be normal variability of diameter size about Z, so that the observed fibre diameter, Y, would have the distribution

$$Y|Z \sim N(z,\sigma^2)$$

whence $$f_Y(y) = \frac{\theta}{\sqrt{2\pi}} e^{-\theta y+\sigma^2\theta^2} \int_{\frac{-(y-\sigma^2\theta)}{\sigma}} e^{-\frac{1}{2}z^2} dz$$

and $$Var[Y] = \sigma^2+\theta^{-2}$$

and $$E[Y] = \theta^{-1}$$

This is a long-tailed distribution arising from a very simple model. If Var [Y|Z=z] also depends on Z, a bimodal sample could readily arise. However, Nagorcka's work makes the basic model appear most unlikely. What is needed is a one-dimensional model arising from Nagorcka's elucidation of the process of follicle formation.

INHERITANCE OF THE STATISTICS OF THE DISTRIBUTION

In the absence of a satisfactory fundamental model, we can continue to work with the statistics of the empirical distribution, but take advantage of new measurement technology to extend the range of statistics used. The Sirolan Laserscan™ rapidly and cost-effectively provides the data from which measures of skewness and kurtosis may be calculated. Most FDD are leptokurtic and long tailed to the right. Table 3 shows heritabilities and genetic correlations for the mean, variance, skewness and kurtosis (Fisher's g_1 and g_2) of FDD from fine wool Merinos in CSIRO's quality flock.

Table 3 Preliminary estimates of genetic parameters of FDD parameters in 9 fine wool bloodlines (genetic correlations in lower triangle, phenotypic in upper, estimates ± standard error when shown).

	Mean FD	Variance FD	SDFD	Skewness	Kurtosis
Phenotypic variance	1.09	4.75	0.12	0.48	83.81
Heritability	0.61 ±0.10	0.37 ±0.08	0.38 ±0.08	0.10 ±0.05	0.14 ±0.05
MFD	-	0.25	0.26	-0.05	-0.03
VFD	0.36 ±0.13	-	0.99	0.12	-0.03
SDFD	0.38 ±0.13	0.99 ±0.00	-	0.12	-0.03
Skewness	0.00 ±0.22	-0.39 ±0.22	-0.49 ±0.22	-	0.93
Kurtosis	0.02 ±0.20	-0.66 ±0.20	-0.72 ±0.19	0.90 ±0.06	-

From these very preliminary measurements, it appears likely that one could select simultaneously for reduced fibre diameter, reduced skewness, and constant (high) kurtosis, a desirable distribution shape more characteristic of good fine wool than of medium wool. The CSIRO fine wool experiment will allow these tentative inferences to be tested fully, after which recommendations can be made about use of the traits discussed here in selection indexes.

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