ITERATIVE CALCULATION OF INBREEDING AND RELATIONSHIP COEFFICIENT*

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SUMMARY

Formulas are proposed for iterative calculation of inbreeding and relationship coefficients. With the formulas, inbreeding and relationship coefficients of offsprings can be calculated on those of individuals in parent generation. The formulas are not only used as general form of iterative formulas for regular pedigrees, but also suitable for any irregular pedigrees without generation overlap.

INTRODUCTION

It is convenient to calculate coefficients of inbreeding and relationship with iterative method in avoidance of path analysis required by the Wright's formula. The following formula established by S. Wright, J. Haldant and R. Fish et al are commonly used for specific regular mating system.

1. Gull–sib mating system

$$f_t = \frac{1}{4}(1 + 2f_{t-1} + f_{t-2})$$

2. Half–sib mating system

$$f_t = \frac{1}{8}(1 + 6f_{t-1} + f_{t-2})$$

3. Self mating system

$$f_t = \frac{1}{2}(1 + f_{t-1})$$

This paper establishes a iterative method for calculating inbreeding and relationship coefficients. The formula can be used not only as a general form of the formulas above, but also suitable for any irregular mating system required by iterative method.

MODEL PEDIGREE

With iterative method, inbreeding and relationship coefficient are calculated from generation to generation. Generation overlap can not be used for iterative calculation. A model pedigree is showed in Fig 1.

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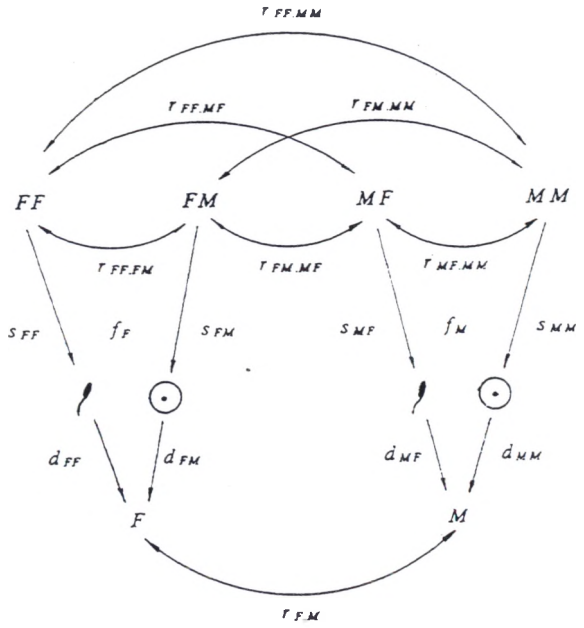

Fig 1 Model pedigree for iterative calculation

FF and FM are of parents of F. MF and MM are parents of M. f_i stands for inbreeding coefficient of individual i, $r_{i,j}$ for relationship coefficient between individual i and j, s_i for path coefficient from individual i to its gamete, d_i for path coefficient from gamete of individual i to its offspring.

The relation between path coefficient from parent p to its gamete (s_p) and inbreeding coefficient of parent p (f_p), the relation between path coefficient from gamete of parent p to its offspring(d_p) and inbreeding coefficient of offspring (f_o) are known commonly as:

$$s_p = \sqrt{\frac{1+f_p}{2}}$$

$$d_o = \sqrt{\frac{1}{2(1+f_o)}}$$

ITERATIVE FORMULA

If the inbreeding and relationship coefficients of F and M can be calculated on those of FF, FM, MF, MM, the iterative calculation will be realized.

Based on path relation Fig 1, Two formulas were proposed in this paper.

1. Formula for inbreeding coefficient

For the inbreeding coefficient of F, the following formula is proposed:

$$f_F = s_{FF}s_{FM}r_{FF,FM} = \sqrt{\frac{1+f_{FF}}{2}}\sqrt{\frac{1+f_{FM}}{2}}r_{FF,FM} = \frac{1}{2}\sqrt{(1+f_{FF})(1+f_{FM})}r_{FF,FM}$$

Inbreeding coefficient of M is calculated in same form:

$$f_M = \frac{1}{2}\sqrt{(1+f_{MF})(1+f_{MM})}r_{MF,MM}$$

With this formula, inbreeding coefficient of offspring can be calculated on inbreeding and relationship coefficients of the individuals in parent generation.

2. Formula for relationship coefficient

For the relationship coefficient between F and M, the following formula is proposed.

$$\begin{aligned}
r_{FM} &= s_{FF}d_{FF}s_{MF}d_{MF} + s_{FF}d_{FF}s_{MM}d_{MM} + s_{FM}d_{FM}s_{MF}d_{MF} + s_{FM}d_{FM}s_{MM}d_{MM} \\
&= \sqrt{\frac{1+f_{FF}}{2}}\sqrt{\frac{1}{2(1+f_F)}}\sqrt{\frac{1+f_{MF}}{2}}\sqrt{\frac{1}{2(1+f_M)}}r_{FF,MF} + \\
&\sqrt{\frac{1+f_{FF}}{2}}\sqrt{\frac{1}{2(1+f_F)}}\sqrt{\frac{1+f_{MM}}{2}}\sqrt{\frac{1}{2(1+f_M)}}r_{FF,MM} + \\
&\sqrt{\frac{1+f_{FM}}{2}}\sqrt{\frac{1}{2(1+f_F)}}\sqrt{\frac{1+f_{MF}}{2}}\sqrt{\frac{1}{2(1+f_M)}}r_{FM,MF} + \\
&\sqrt{\frac{1+f_{FM}}{2}}\sqrt{\frac{1}{2(1+f_F)}}\sqrt{\frac{1+f_{MM}}{2}}\sqrt{\frac{1}{2(1+f_M)}}r_{FM,MM} \\
&= \frac{1}{4}\cdot\frac{1}{\sqrt{(1+f_F)(1+f_M)}}\left[\sqrt{(1+f_{FF})(1+f_{MF})} + \sqrt{(1+f_{FF})(1+f_{MM})} + \right. \\
&\left.\sqrt{(1+f_{FM})(1+f_{MF})} + \sqrt{(1+f_{FM})(1+f_{MM})}\right]
\end{aligned}$$

With this formula the kinship between offsprings can be calculated on the inbreeding and relationship coefficients of individuals in parent generation and inbreeding coefficients of offsprings.

DISCUSSION

The formulas for inbreeding and relationship coefficient are widespread for any pedigree without generation overlap. They have also sample form for the regular mating system. When X is assumed to be a offspring of individual F and M and considered to be individual in generation t, Therefore, F and

M are in generation t−1 and FF, FM, MF, MM are in generation t−2. The same form in Section Introduction are got.

1. Full−sib mating system

In Fig 1, When FF is same individual with MF, and FM is same individual with MM, a pedigree of full−sib mating system is formed. In this case, $r_{FF,MF} = r_{FM,MM} = 1$, $r_{FF,FM} = r_{FF,MM} = r_{FM,MF} = r_{MF,MM}$, $f_{FF} = f_{FM} = f_{MF} = f_{MM}$

$$f_F = f_M = \frac{1}{2}(1 + f_{FF})r_{FF,FM}$$

$$r_{F,M} = \frac{1}{2}\frac{1}{1+f_F}(1 + 2f_F + f_{FF})$$

$$f_x = \frac{1}{2}\sqrt{(1+f_F)(1+f_M)}\,r_{F,M} = \frac{1}{4}(1 + 2f_F + f_{FF})$$

2. Half−sib mating system

In Fig 1, When FF is same individual with MF, a pedigree of half−sib mating system is formed. In this case, $r_{FF,MF} = 1$, $r_{FF,FM} = r_{FF,MM} = r_{FM,MF} = r_{MF,MM} = r_{FM,MM}$, $f_{FF} = f_{FM} = f_{MF} = f_{MM}$

$$f_F = f_M = \frac{1}{2}(1 + f_{FF})r_{FF,FM}$$

$$r_{F,M} = \frac{1}{4}\frac{1}{1+f_F}(1 + f_{FF} + 6f_F)$$

$$f_x = \frac{1}{2}\sqrt{(1+f_F)(1+f_M)}\,r_{F,M} = \frac{1}{8}(1 + f_{FF} + 6f_F)$$

3. Self mating system

In Fig 1, When FF is same individual with FM, MF, MM, and F is same individual with M. A pedigree of self mating system is formed. In this case, $r_{F,M} = r_{FF,MF} = r_{FF,FM} = r_{FF,MM} = r_{FM,MF} = r_{MF,MM} = r_{FM,MM} = 1$, $f_{FF} = f_{FM} = f_{MF} = f_{MM}$

$$f_x = \frac{1}{2}(1 + f_M)$$

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