MAXIMIZING UTILITY WHEN SELECTING SIRES GLOBALLY

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SUMMARY

Twenty progeny tested sires from each of two countries - Canada and the US, were used to demonstrate the use of quadratic programming (QP) for maximizing the utility function. Utility functions maximize net present value (NPV) of sire selection decisions considering attitude towards risk. NPV were calculated for each sire based on the semen cost, interest rate and production to type selection policy. NPV expressed the value of genetic contributions to descendants over 3 generations. Estimated transmitting abilities (ETA) of US sires were converted to Canadian equivalents. Selection accuracies on US sires were adjusted to account for accuracy of the conversion formulae. Risk represents the variance in NPV and is a function of selection accuracy. Variances and covariances of sire NPV were considered. Inclusion of covariances based on sire additive genetic relationships had little or no effect on utility functions. Sires included in the portfolio remained the same; however, with increasing aversion to risk, matings to sons of a particular sire decreased when relationships were considered. When aversion to risk increased, the number of sires included in the optimal portfolio increased and sires with high selection accuracy receive increased usage. Associated reductions in risk were up to 47.6% with small (5.2%) reduction in NPV. When interest rates were increased by 4%, NPV and risk decreased by 27.2% and 25.2%, respectively. Increasing selection pressure on type relative to production increased risk by 17.5% with a 19.3% increase in NPV; sire selection and usage changed significantly. The use of QP is an effective tool for sire selection when risk is to be considered. A difficulty arises in generalizing sire usage recommendations because selection decisions are based on personal utility functions.

INTRODUCTION

Dairy farmers are confronted with a complex decision making process when selecting sires to maximize returns over investments. Investment in genetic improvement through sire selection is a long term consideration. Calculation of NPV based on appropriate interest rates, enables ranking of sires who vary in semen price and in ETA for several traits. Canadian dairy farmers have traditionally placed emphasis on both production and type traits when selecting sires. Calculation of NPV is complicated by poor estimates of the economic value for type traits. Widespread use of semen enables access to sires globally. Sire ETA in the exporting country may be converted to ETA equivalents in the importing country. Selection on converted ETA need consider the additional risk (variance of NPV) attributed to the accuracy of the conversion formulae and the fact that genetic correlations between traits in the two countries are <1. Based on sire selection patterns (Leitch, unpublished) Canadian dairy farmers are averse to risk. Portfolio theory simultaneously considers both expected income and risk in maximizing the utility function of the decision maker. The utility function is determined based on the weighting of NPV versus risk. Schneeberger et al. (1982) demonstrated the use of QP to determine the optimal portfolio assuming sires are unrelated. The objectives of this study were to apply QP and study the effect on the optimal portfolio of the following factors: 1) accounting for relationships among sires, 2) degree of aversion to risk, 3) varying interest rate and 4) selection emphasis on type relative to production.

MATERIALS AND METHODS

For purely illustrative purposes, data from 20 Canadian and 20 US proven sires were used. Canadian sires were the top ranking Holstein sires based on the January 1994 lifetime profitability index (LPI) which is a selection tool used by Canadian dairy farmers and combines ETA for production and type traits. US sires represented the top 20 marketed by a leading US semen distributor. US sires were selected on the basis of a LPI, determined from converted ETA for production and type traits.

CONVERTED ETA and ACCURACY US sire ETA for production and type traits were converted to Canadian equivalents according to Robinson (1994) and Lohuis (1994). Squared selection accuracy of the US ETA ($Rept_{k\ USj}$) of the *j*th sire was converted to Canadian values ($Rept_{k\ CANj}$) for *k*=milk or final class by accounting for the accuracy of the conversion formula ($R^2_{k\ CAN,US}$).

$$Rept_{k_{CANj}} = Rept_{k_{USj}} R^2_{k_{CAN,US}} \qquad (1)$$

MAXIMIZING UTILITY Quadratic programming (QP) was used to find the NPV vs variance frontier (minimum variance for each NPV) following the approach described by Schneeberger et al. (1982). In matrix notation the QP problem may be written as;

$$\textit{Maximize } U = p'v + lv'Qv \qquad \textit{Subject to } A\ v \leq b \textit{ and } v \geq 0$$

where U is the utility, *v* is the vector of solutions such that the *j*th element is the proportion of cows in the herd that is to be bred to sire *j*, *p* is a vector of NPV for sires [2], , *A* is a coefficient matrix of constraints, *b* is a vector of right-hand sides of constraints, and *l* is the subjective weight for variance relative to expected income and describes attitudes to risk, with possible values from -1 (averse) to 1 (prone); *Q* is a matrix of variances [6] and covariances [7] of NPV among sires. NPV was modelled for the Canadian situation based on the NPV of successfully producing a replacement;

$$NPV_j = -nP_j + B(MT\$_j)(1+F_G) \qquad (2)$$

where NPV_j is the net present value of genetic contribution to all descendants (i.e., total worth of gene flow) of sire *j*; P_j is the price per unit of semen; n (=4) is the number of units of semen used to produce a female offspring;

$$MT\$_j = -\$1.5MILK_j + \$5.5FAT_j + \$6.6PROTEIN_j + w[(.3FC_j + .4MS_j + .2FL_j + .1CP_j)/\sigma_T]\sigma_{M\$} \qquad (3)$$

is the net income from production and type per lactation due to genetic change for a daughter of sire *j*, *w* is the weighting on type relative to a weighting of 1.0 on production; σ_T (=3.4) is the standard deviation of composite type traits, FC_j, MS_j, FL_j and CP_j which denote ETA for final class, mammary system, feet and legs and capacity; $\sigma_{M\$}$ (=30.6) is standard deviation of the composite production traits;

$$B = 3(1+i)^{-[(t-1)(c/12)+y]} \qquad (4)$$

is the discounting factor for net income from production and type of an average daughter for an average lifetime of *t*(=3) lactations; *i* is the real discount rate; *c* (=13) is the calving interval in months; *y* (=3.1) is the number of years after first breeding the dam when the replacement daughter is at the mid income point of the first lactation,

$$F_G = \sum_{g=1}^{G} 2^{-g}(1+i)^{-gh} \qquad (5)$$

is the discounting factor for expected net income increases from *G* (=2) additional generations (grand and great-grand daughters of sire *j*) of descendants; *g* is generation number of descendant; *h* (=4.9) is the average age at which a replacement female calf is born to a dam; 2^{-g} accounts for the decay in net income due to Mendelian sampling from parent to offspring and to subsequent descendants. The matrix *Q* contains the estimate error variances and covariances of NPV_j determined as

$$V(NPV_j)=[B(1+F_G)]^2(1-R_j)\sigma^2_{MT\$} \quad (6)$$

$$Cov(NPV_j NPV'_j)=a_{jj'}\sigma^2_{MT\$}[B(1+F_G)]^2\sqrt{(1-R_j)(1-R'_j)} \quad (7)$$

where $\sigma^2_{MT\$}$ (=1196 or 1876 for a 2:1 or 1:1 selection policy) is the variance of MT$; $a_{jj'}$, is the additive genetic relationship (ignoring inbreeding) between sire *j* and *j'*; and R_j is the average weighted repeatability of milk ($R_{Milk\ j}$) and final class ETA ($R_{Final\ class\ j}$) for the *j*th sire:

$$R_j=(R_{Milk_j}+wR_{Finalclass_j})/(1+w) \quad (8)$$

RESULTS and DISCUSSION

Table 2 presents the portfolios and utility functions for the different situations described in Table 1. Sires are listed in Table 2 in order of their NPV for situation 1 described in Table 1. Due to space limitations only 16 bulls are shown. When aversion to risk increased, the number of sires included in the optimal portfolio increased. At the highest level of risk aversion considered, there were an increased number of matings to bull #7 who had a high accuracy of selection. Associated reductions in risk were up to 47.6% with small (5.2%) reduction in NPV. Inclusion of relationships had no effect on the portfolio, risk or NPV. With increasing risk aversion, inclusion of relationships had little effect on the utility function and the sires included in the portfolio remained the same; however, matings to sons of a particular sire decreased when relationships were considered. A negative value (aversion) for the risk coefficient was used, based on results of Schneeberger et al. (1981). All decision makers may not be averse to risk. Woolliams and Meuwissen (1993), using a Bayesian approach, developed a framework for considering the rationality of propensity for risk in the context of a breeding scheme and showed that genetic response is maximized when risk is favoured. When interest rates were increased by 4%, NPV and risk decreased by 27.2% and 25.2%, respectively. Increasing selection pressure on type relative to production increased risk and NPV by 17.5% and 19.5%, respectively. Sire selection and usage rate changed significantly. Risk increased as a function of the lower selection accuracies for type and the increased variance with increased weight on type. The assignment of an economic value for type traits has been treated subjectively in this study and determination of the true economic value for type traits, and other workability traits such as calving ease, milking speed, temperament and health would increase the usefulness of utility theory. US bulls were excluded from portfolios due to the high price of semen: average semen prices were 46% higher than for the average of Canadian sires (excluding bull #6 whose price was artificially high due to poor semen production related to injury). By reducing semen prices for US sires by 46%, US bulls were included in the optimal portfolio for the situation where aversion to risk is highest. At first glance it may appear counter-intuitive to increase matings to sires who represent additional risk based on their converted ETA. This risk is countered by including an increased number of bulls in the optimal portfolio. This study dealt only with sire selection and ignores consideration of relationships within the herd to be mated. Additional constraints may be imposed to limit the usage of any particular bull in a herd.

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Table 1. Summary of situations considered

Situation	Ratio of emphasis Production:Type (w)		Interest rate (i)	Risk coefficient (l)	Relationships between sires considered
1	2:1	w=.50	3%	-.02	yes
2	2:1	w=.50	3%	-.04	yes
3	2:1	w=.50	3%	-.06	yes
4	2:1	w=.50	3%	-.06	no
5	2:1	w=.50	7%	-.02	yes
6	1:1	w=1.0	3%	-.02	yes
7[1]	2:1	w=.50	3%	-.06	yes

[1] Semen price on US bulls reduced by 46%.

Table 2. Optimal usage rates (solutions to quadratic programming problem) of sires within a herd based on situations described in Table 1.

Bull Id.[1,2]	Sire Id.	Milk ETA	Fat ETA	Prot. ETA	Type[3] ETA	Rept[4] Milk	Rept[4] Type	Semen Price	1	2	3	4	5	6	7
1 Cdn	A	18	22	20	10	90	72	35	80.4	49.5	35.9	34.4	79.6	56.5	35.5
2 Cdn	B	12	19	19	4	90	79	10	11.9	16.9	14.6	12.9	14.7		14.2
3 Cdn	C	4	21	14	6	90	78	10	5.2	10.1	7.9	10.4	5.7		7.8
4 Cdn	D	16	24	16	5	94	73	18	2.6	12.8	12.0	10.2			11.6
5 Cdn	C	9	22	15	6	88	76	15		5.1	4.2	7.5			4.2
6 Cdn	C	21	30	21	14	90	75	150		5.6	4.7	8.0		43.5	4.6
7 Cdn	C	21	13	24	9	99	98	30			20.8	16.6			13.5
8 Cdn	E	20	21	19	2	85	64	15							
9 Cdn	A	17	20	18	3	88	76	15							
10 Cdn	F	13	23	16	6	83	70	50							
11 US	G	19	20	20	6	80	71	48							5.1
12 US	G	20	21	20	3	68	61	35							2.3
13 US	H	19	17	22	2	73	62	26							
14 Cdn	A	11	17	14	9	89	77	15							
15 US	I	19	13	26	3	77	67	38							
16 US	A	22	18	22	4	72	63	45							1.2
Utility of optimal solution									962	934	918	920	708	1141	920
Expected net present value									$1006	974	954	953	732	1202	954
Risk (variance of expected net present value)									$46.8	31.8	24.5	23.3	35.0	55.1	24.3

[1] Bulls are listed in order of NPV$ for situation 1 described in Table 1.
[2] Cdn = Canadian proven sire, US = US proven sire
[3] Type = .3 final class + .4 mammary system + .2 feet and legs + .1 capacity
[4] Repeatability = selection accuracy^2